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Confidence Interval Analysis of Dynamic Performance of 3RRR Manipulator Considering Parameters with Geometric Uncertainty

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Abstract: *Uncertain geometry may have great influence on the performance of kinematic and dynamic of robotic manipulators. Following statistical concepts, one can make intelligent judgments and right decisions when a system presents uncertainties and variations. Therefore, statistical methods can be used to evaluate the error caused by manufacturing processes of robotic components. In this paper, considering the tolerance of manufacturing processes, a normal distribution is applied in order to evaluate the impact of uncertainties on the geometric parameters of the manipulator 3RRR. This impact is assessed by modeling the inverse kinematic and dynamic models of the manipulator 3RRR, considering the confidence interval until 95% of the proposed parameters. This analysis is performed using MATLAB®. Moreover, the presented technique allow the calculation of a reliable range of efforts that the manipulator is subject to perform a desired task.*
Keywords: *parallel kinematic manipulator, confidence interval, kinematic and dynamic analysis and geometric uncertainty*

1. INTRODUCTION

Uncertain parameters in robotics may cause errors in the end-effector's position, high dynamic efforts when the end-effector is near of an unexpected singularity, unstable control, among other drawbacks. For that reason, it is important to analyze the sensitivity of manipulators' performance to variations in their geometric parameters. Parallel manipulators have gained the attention of the research community in the last years due to their promising capabilities when compared to serial manipulators. Although parallel manipulators have been widely studied, papers about their dynamic performance considering their parameters' variations are not well explored.

Some studies have proposed different strategies to control manipulators with uncertain parameters. Li *et al.* (2001) have proposed Fuzzy P+ID control which consists on a fuzzy logic proportional controller and a conventional integral and derivative controller. This technique has been implemented to control an EDDA manipulator and it proved to be more efficient when compared to the classical PID. Song *et al.* (2005) have presented a computed torque control with a compensator considering uncertain parameters using fuzzy logic. Kim (2004) has proposed an adaptative control method applying fuzzy logic. It has provided a systematic methodology to design output feedback control for robotic systems with uncertain parameters.

Moreover, regarding the accuracy of manipulators, Binaud *et al.* (2010) have presented a study about the sensitivity of the pose of parallel manipulators to variations on their parameters. Tsai and Lai (2004) have studied the transmission quality of manipulators with joint clearance. Also, Tsai and Lai (2008) have made an accuracy analisys of multi-loop mechanisms treating the joint clearances as virtual links.

The aforementioned studies have presented efficient methods to control manipulator with uncertain models and different strategies to analyze their accuracy considering joint clearance. However the dynamic performance of the presented studies has not been considered. Regarding errors caused by manufacturing processes, this paper proposes a method in order to infer about the impact on the dynamic performance of parallel manipulators to variantions on their geometric parameters. As a case study, a 3RRR manipulator (Fig: 1) is taken as example.

The variation of parameters is performed considering manufacturing tolerance ISO 286-1: 2010 (E), based on the degree of tolerance for IT16. The degree of tolerance IT16 was chosen to be an overestimation of the error, i.e. the studied case is treated as the worst one. Given this possible variation on the links of the manipulator, a confidence interval

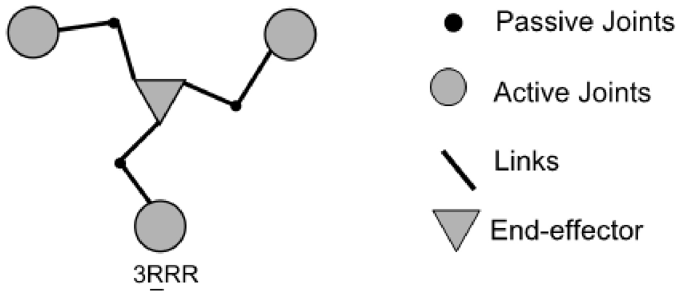

Figure 1: **:Planar parallel manipulator 3RRR.**

range until 95% of these values are considered. This confidence interval is based on a Gaussian probability distribution of the possible size of the manipulator link, taking the average size of this data at the nominal value of 200mm and standard deviation as the manufacturing tolerance given by ISO 286-1: 2010 (E), considering IT16.

The numerical methodology employed to model the 3RRR manipulator is briefly described in Section 2. Some concepts regarding Interval Analysis are treated in Section 3. Numerical results are presented in Section 4. Finally, in Section 5, conclusions based on these numerical results are drawn.

2. Modelling

Figure 2 shows the linear and angular positions of the manipulator 3RRR used for modeling the kinematics and dynamics. The positions are given by x, y and α, respectively, the first two are linear positions and the last is angular position of the end-effector. Since the angular positions of joints A_i and B_i are represented respectively by θ_i, β_i, with kinematic chain i corresponding to 1, 2 or 3. The angular position of the joint C_i is represented by the angle formed between the horizontal and the imaginary line between the centroid of the end-effector to the joint study, and this distance is given by h. The links l_1 and l_2 are presented.

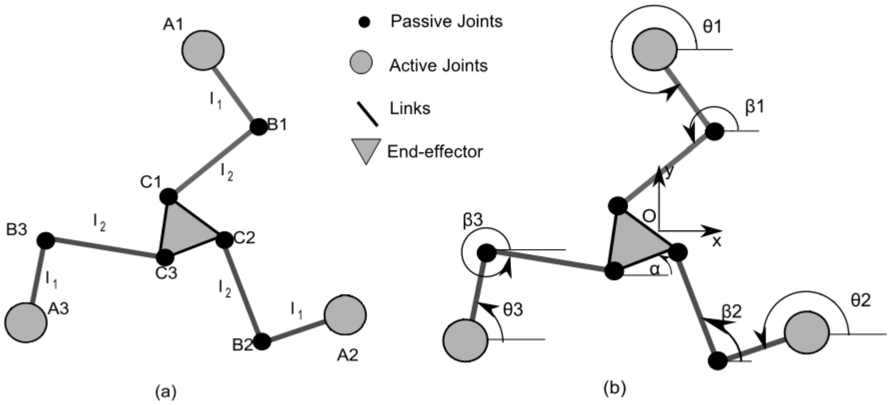

Figure 2: **:3RRR geometrical characteristics: (a) length of the links and (b) joint angles.**

2.1 Inverse Kinematic

The inverse kinematics is evaluated in order to calculate the angles θ_i and β_i for a given position of the end-effector. This can be done by evaluating some geometrical relationships related to the links l_1 and l_2 of the manipulator and the position of the joints and the centroid of the end-effector (Fig. 2). The angle represented by $\alpha + \lambda_i$ is the angle between the horizontal of the joint C_i tangent to the imaginary line that connects this vertex to the centroid of the end-effector.

The position of C_i can be written by:

$$\mathbf{r_{C_i}} = \mathbf{r_{B_i}} + l_2 \begin{bmatrix} cos(\beta_i) \\ sin(\beta_i) \end{bmatrix} \tag{1}$$

were $\mathbf{r_{B_i}} = (\mathbf{x_{B_i}}, \mathbf{y_{B_i}})$ and $\mathbf{r_{C_i}} = (\mathbf{x_{C_i}}, \mathbf{y_{C_i}})$. with this equation, one can define:

$$\|\mathbf{r_{C_i}} - \mathbf{r_{B_i}}\| = l_2 \tag{2}$$

Using the definition of (Fontes *et al.*, 2014), define μ_i e ρ_i (Eq. 3):

$$\begin{bmatrix}\mu_i\\ \rho_i\end{bmatrix} = \begin{bmatrix}x\\ y\end{bmatrix} + h\begin{bmatrix}cos(\alpha+\lambda_i)\\ sin(\alpha+\lambda_i)\end{bmatrix} - \begin{bmatrix}x_{0_i}\\ y_{0_i}\end{bmatrix} \quad (3)$$

$$\left\|\begin{bmatrix}\mu_i - l_1 cos(\theta_i)\\ \rho_i - l_1 sin(\theta_i)\end{bmatrix}\right\| = l_2 \quad (4)$$

Equation 4 describes the following geometrical constraint:

$$-2l_1\rho_i sin(\theta_i) - 2l_1\mu_i cos(\theta_i) + \mu_i^2 + \rho_i^2 + l_1^2 - l_2^2 = 0 \quad (5)$$

In order to describe the angles θ_i, one can define:

$$e_{i1} = -2l_1\rho_i \quad (6)$$

$$e_{i2} = -2l_1\mu_i \quad (7)$$

$$e_{i3} = \mu_i^2 + \rho_i^2 + l_1^2 - l_2^2 = 0 \quad (8)$$

Using these definitions, the angles θ_i can be calculated:

$$\theta_i = 2\tan^{-1}\left(\frac{-e_{i1} \pm \sqrt{e_{i1}^2 + e_{i2}^2 - e_{i3}^2}}{e_{i3} - e_{i2}}\right) \quad (9)$$

Based on the angles θ_i, one can calculate the angle β_i through the Eq. (3):

$$\begin{bmatrix}\mu_i - l_1 cos(\theta_i)\\ \rho_i - l_1 sin(\theta_i)\end{bmatrix} = l_2\begin{bmatrix}cos(\beta_i)\\ sin(\beta_i)\end{bmatrix} \quad (10)$$

$$\beta_i = \tan^{-1}\left(\frac{\rho_i - l_1\sin(\theta_i)}{\mu_i - l_i\cos(\theta_i)}\right) \quad (11)$$

2.2 Dynamic

The inverse dynamics is exploited in order to evaluate the required actuator's torque to perform a pre-defined trajectory (end-effector's position, velocity and acceleration). To this end, the Jacobian is firstly defined. This matrix described the relation between the actuators' angular velocities and end-effector's velocities. This relation is defined by:

$$\dot{\mathbf{X}} = J\dot{\mathbf{\Theta}} \quad (12)$$

with $\mathbf{X} = \begin{bmatrix}x & y & \alpha\end{bmatrix}^T and \mathbf{\Theta} = \begin{bmatrix}\theta_1 & \theta_2 & \theta_3\end{bmatrix}^T$. The link A_iB_i is rotating around the joint A_i, while the link B_iC_i presents rotational and translational movements. Considering these aspects, the Principle of Virtual Work is exploited to evaluate the required torque for a given set of position, velocities and accelerations. Details can be found in Fontes *et al.* (2014). In order to summarize the technique, the vectors and can be defined. They are composed by forces (F) and moments (M) applied to the body j of the chain i:

$$\mathbf{p_{ij}} = \begin{bmatrix}\mathbf{F_{ij}}\\ M_{ij}\end{bmatrix} \quad (13)$$

where $j = 1$ is the link A_iB_i and $j = 2$ is the link B_iC_i, and to the end-effector:

$$\mathbf{p_{ee}} = \begin{bmatrix}\mathbf{F_{ee}}\\ M_{ee}\end{bmatrix} \quad (14)$$

Both p_{ij} and p_{ee} can be rewritten according to the variation of the linear position, $r_{x_{ij}}$ and $r_{y_{ij}}$, and its angular position $\phi_{ij} = \begin{bmatrix}\Theta_i & \beta_i & \alpha\end{bmatrix}$:

$$\mathbf{p_{ij}} = Z_{ij}\begin{bmatrix}r_{\ddot{x}_{ij}}\\ r_{\ddot{y}_{ij}}\\ \ddot{\phi}_{ij}\end{bmatrix} + Nij\begin{bmatrix}r_{\dot{x}_{ij}}\\ r_{\dot{y}_{ij}}\\ \dot{\phi}_{ij}\end{bmatrix} \quad (15)$$

$$\mathbf{p_n} = Z_n \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\alpha} \end{bmatrix} \tag{16}$$

where:

$$Z_{ij} = \begin{bmatrix} m_j & 0 & -m_j s_{ij} \sin \phi_{ij} \\ 0 & m_j & m_j s_{ij} \cos \phi_{ij} \\ -m_j s_{ij} \sin \phi_{ij} & m_j s_{ij} \cos \phi_{ij} & I_j \end{bmatrix} \tag{17}$$

$$N_{ij} = \begin{bmatrix} 0 & 0 & -m_j \dot{\phi_{ij}} s_{ij} \cos \phi_{ij} \\ 0 & 0 & -m_j \dot{\phi_{ij}} s_{ij} \sin \phi_{ij} \\ 0 & 0 & 0 \end{bmatrix} \tag{18}$$

and is the inertia of the body j, and is m_1, m_2 and $m_e e$ are the masses of each link , and the end-effector, respectively. The required torque to perform a predefined trajectory is:

$$\boldsymbol{\tau} = M\ddot{\Theta} + V\dot{\Theta} \tag{19}$$

where:

$$M = (J^t Z_n J + \sum_{i=1}^{3} \sum_{j=1}^{3} K_{ij}^t Z_{ij} K_{ij}) \tag{20}$$

$$V = (J^t Z_n \dot{J} + \sum_{i=1}^{3} \sum_{j=1}^{3} K_{ij}^t Z_{ij} \dot{K_{ij}} + \sum_{i=1}^{3} \sum_{j=1}^{3} K_{ij}^t N_{ij} K_{ij}) \tag{21}$$

The matrices K are defined in Fontes *et al.* (2014).

3. Interval Analysis

3.1 Probability for Engineering

Statistical concepts and methods often means that we understand more the world around us. They provide ways of gaining new insights into the behavior of many phenomena that you will encounter in your chosen field of specialization in engineering or science (Devore, 2015). With the statistics can make informed decisions and smart judgments under uncertainty and variations. Physical uncertainty can be described in terms of probability distribution, resulting from physical observation regarding randomness (Noh *et al.*, 2011). These variations and uncertainties motivate the use of statistics, with the aim of achieving a standard for the data in the study (Devore, 2015). To evaluate the torque required to move the effector during the course, the confidence interval method is used, and is considered a reliability of 95% of all possible values for the length of the links. As for the distribution of sizes of the links is considered a Gaussian probability distribution.

3.2 Gaussian Distribution of Probability

An important feature of industrial robots is their ability to reach a certain position with the lowest possible error. A common method to measure this error is finding its accuracy and repeatability. Many of these methods are similar, they can be approximated to a normal distribution as the probability is calculated (Ramsli, 1988). The accuracy of this approach depends on the degree of repeatability that follows the Gaussian distribution. Ramsli (1988) showed that the studied industrial robots, considering 100 samples, only those with negative skewed distribution can not be approximated to the normal distribution. Thus, this study is given to manufacturing processes that do not follow an negative skewed distribution of probabilities. In normal probability distribution have σ as standard deviation and ρ as the average, thus have the probability density function (probability density function, PDF) is given by:

$$\mathbf{f}(\mathbf{x}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\rho)^2}{2\sigma^2}} \tag{22}$$

with $f(x)$ being the possibility of the respective length x of the link.

3.3 Confidence Interval

For the confidence interval, a range of 95% reliability was considered. For this, we need to make a setting to facilitate the calculation, given by: $\sigma = 1$, and $\rho = 0$, so $f(x)$ is represented by:

$$\mathbf{f}(\mathbf{x}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{23}$$

Another definition that is required to be made Z is given by:

$$\mathbf{Z}(\mathbf{x}) = \frac{x - \rho}{\sigma} \tag{24}$$

And considering σ values and μ defined, it $Z = x$ and considering the standard cumulative normal table of probability, the Z value for the area of the normal probability density function is found. With Z to this interval, the value of x corresponding to Z is given by:

$$\mathbf{x} = Z\sigma + \rho \tag{25}$$

Thus the corresponding values limite are 2.5% up to 97.5% are found. Note that the area of the function between these ranges is given by 95% of confiability.

4. Results

The 3<u>R</u>RR manipulator is shown in fig. 3. The trajectory (in red) and the respective positions of the actuators 1, 2 and 3 as $(-0.17, -0.10)$, $(-0.17, 0.10)$ and $(0.00, 0.20)$. The distance between the vertices of the effector to its centroid is given by $h = 50$mm. The possible size of the length of the links that were studied are shown in table 1.

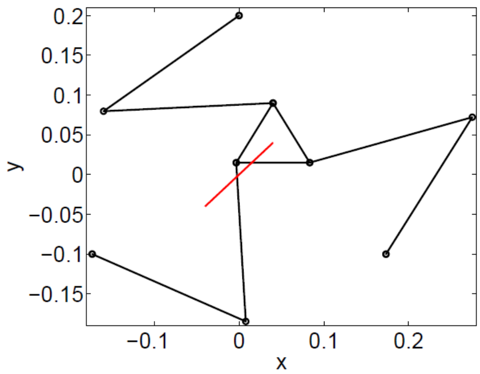

Figure 3: **<u>3</u>RRR and the trajectory (in red)**

The angle α which relates the angular position of the effector is studied in two different ways: the first is made considering this angle being $\alpha = 0$, while the second approach is made with initial $\alpha = -\pi/8$ and the final $\alpha = \pi/8$. Note that the mean (ρ) is represented by the nominal value of the link.

Table 1: **Length of the 3<u>R</u>RR components, the nominal values and their respective interval values.**

Parameters	Nominal Value (m)	Interval Value (m)
l_1 and l_2	0.2	[0.19565, 0.20435]

The geometric variation is performed within that range of values, where the study is done with nine values given in table 2 and their respective probability values are considered between the two values. And fig. 4 represents the probability density function. The colors of filled areas are specified in the table 2.

Note that the area of the shades of blue together are 95% of the possibilities, and consequently the confidence interval are all values between these colors. The values representing the limits of the confidence interval were found from Eq. 24. Replacing the values found in Z table of standard cumulative normal distribution probability corresponding to 2.5% and 97.5% of the total area. These values are 1.96 and -1.96 respectively, replacing in Eq. 25, these values given in Table 2 are found.

From these values, the required torque to do the trajectory of the end-effector are calculated. Two case studies are considered.

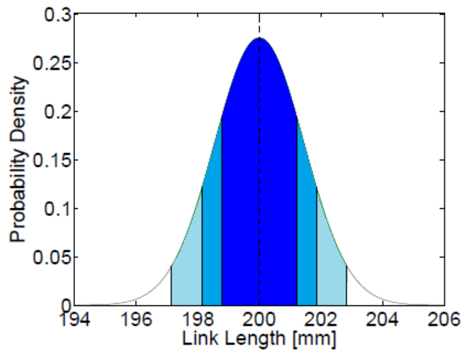

Figure 4: **Probability Density Function of the link length.**

Table 2: **Variation on the nominal value and the probability between these values.**

Area Color	Nominal Value variation (mm)	Probability between these lines (%)
blue	$\pm$ 1.2350	60
blue + turquoise	$\pm$ 1.8582	80
blue + turquoise + light turquoise	$\pm$ 2.8420	95

4.1 The influence of the variation on the length of the links with alpha constant

As aforementioned, in order to assess the influence of the length of the links' variation on the dynamic performance of the manipulator, the required torques to perform the selected predefined trajectory are compared . The columns in Fig.5 depict the required torque for each actuator denoted as 1, 2 and 3 according to the degrees of freedom $\mathbf{\Theta} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \end{bmatrix}^T$. The lines (a) and (b) depict the required torque for each interval values of the evaluated lengths l_1 and l_2 respectively. And the grafics further below (Z) are the representation of the blue square of the respective grafic.

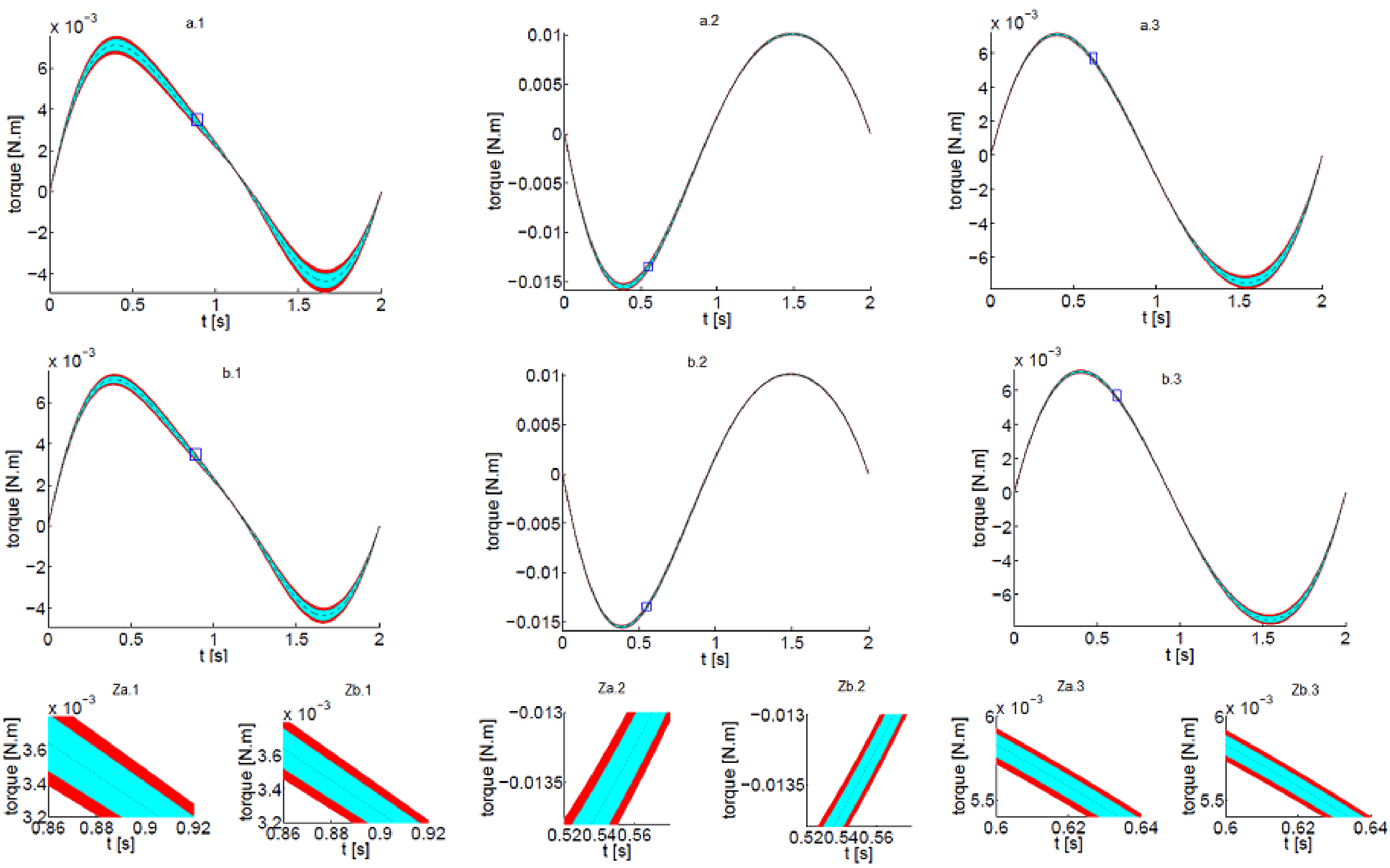

Figure 5: **Interval values of the required torque to perform the pre-defined trajectory regarding the manipulators' actuators (columns** 1, 2 **and** 3**), for** $\alpha = 0$**: (a)** l_1 **and (b)** l_2**. The letter Z indicates zoom on the blue square.**

The region filled by cyan represents the 95% of the reliability, the red region is the rest of the possibilities, and the nominal value of the links is represented by a dashed line. Figures representing the blue square has aimed to get a better graphics display.

According to Figs.5 (a1, a2 and a3, and their respctive Z), one can conclude that variations on the length l_1 have more influence on the dynamic performance of the manipulator than variations on the length l_2 . This is not an intuitive result.

4.2 The influence of the variation on the length of the links with alpha varying

In this case, the required torque to perform the trajectory are compared considering geometric variations and also variation on the orientation angle of the end-effector. The columns in Fig.6 depict the required torque for each actuator denoted as 1, 2 and 3 according to the degrees of freedom $\mathbf{\Theta} = \begin{bmatrix}\theta_1 & \theta_2 & \theta_3\end{bmatrix}^T$. The lines (a) and (b) depict the required torque for each interval values of the evaluated lengths l_1 and l_2 respectively. And the grafics further below (Z) are the representation of the blue square of the respective grafic.

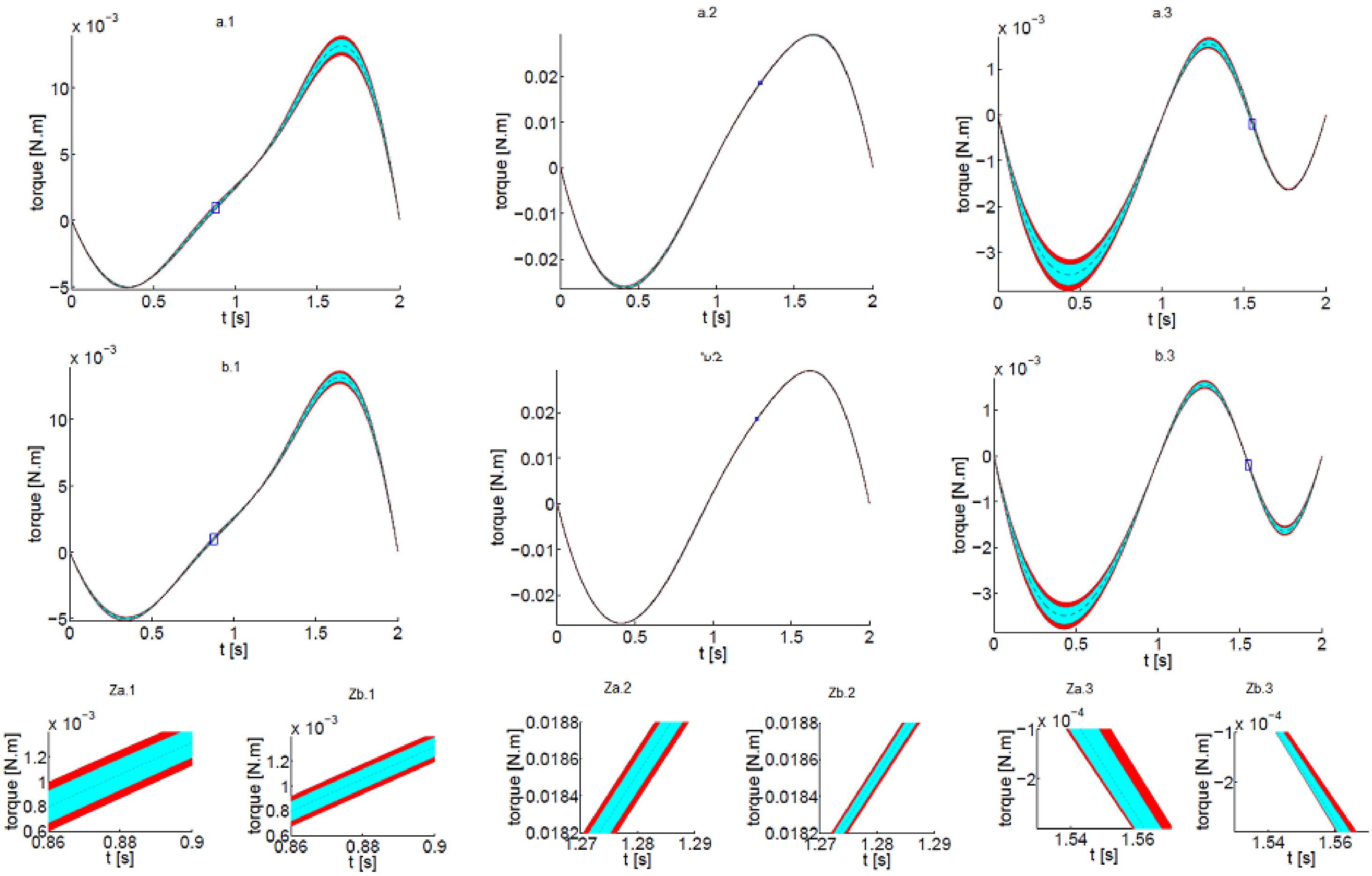

Figure 6: **Interval values of the required torque to perform the pre-defined trajectory regarding the manipulators'actuators (columns 1, 2 and 3), for $\alpha = [-\pi/8, \pi/8]$: (a) l_1 and (b) l_2. The letter Z indicates zoom on the blue square.**

The region filled by cyan represents the 95% of the reliability, the red region is the rest of the possibilities, and the nominal value of the links is represented by a dashed line. Figures representing the blue square has aimed to get a better graphics display.

According to Figs. 6 (a1, a2 and a3, and their respctive Z), one can conclude that variations on the length l_1 have more influence on the dynamic performance of the manipulator than variations on the length l_2 . This is not an intuitive result.

5. Conclusion

- Variations on the length of the link l_1 have slight more influence on the dynamic performance of the manipulator than variations on the length of the link l_2;
- variation of the angle alpha causes the maximum required torque be higher;
- flexibility studies in the links can be important to assess more accurately;

- the intervals of both probability density function as the torque followed the same tendency, where the nominal value of the length is the center of curves and when it departs from that point the function departs proportionately.

The impact of some uncertainties could be measured using interval analysis, dynamic parametric models of the manipulator and statistic. This combination is a straightforward methodology that can aid the predict possible behavior of some robotic components considering manufacture tolerance.

6. Acknowledgment

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7. References

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